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Production Efficiency of a Public Firm, Urban Structure, and Market Size in a General Equilibrium Model

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Abstract

This paper presents a general equilibrium model that integrates a mixed duopoly market into an open monocentric city framework to examine how the production efficiency of the public firm affects the urban structure, market outcomes, and social welfare. We derive closed-form equilibrium solutions and show that a decline in the production efficiency of the public firm contracts the equilibrium city size and market demand, whereas its welfare effect is generally ambiguous because of the trade-off between direct market distortions and indirect effects through the urban structure. Using numerical simulations, we further demonstrate that although social welfare initially decreases as the production efficiency of the public firm deteriorates, it eventually increases once welfare gains from the reduction in total commuting costs and external urban costs dominate the negative market size effect. This qualitative result is robust across different values of the outside option.

Keywords: Mixed Duopoly; Urban Structure; Production Efficiency of the Public Firm; General Equilibrium; City Size.
JEL Classification:R13; L32; D43.

1 Introduction

This paper presents a general equilibrium model that integrates a mixed oligopoly market with the urban structure and theoretically examines the effects of the production efficiency of a public firm on city size, market prices, and social welfare. Specifically, we introduce a monocentric city model into a mixed duopoly framework and explicitly analyze the mechanism through which the production

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decisions of public and private firms influence market size and social welfare through endogenous changes in the urban population.

The traditional urban economics models developed by Alonso (1964), Mills (1972), and Muth (1968)—collectively known as the monocentric city model—describe an urban structure in which households reside in locations surrounding the central business district (CBD) and commute to the CBD for work. However, these models focus on the mechanisms determining the urban structure and land rent and often adopt highly simplified representations of household income and production. As households primarily commute to the CBD to supply labor and earn wage income, incorporating production activities into the monocentric city model is essential for analyzing the urban structure in greater detail.

Numerous industrial organization studies have analyzed various types of market structures, including monopoly, quantity and price competition under oligopoly, and monopolistic competition, to explain economic activities that cannot be adequately captured by the theory of perfect competition. Mixed oligopolistic markets have recently attracted particularly strong attention. For example, De Fraja and Delbono (1989) developed a mixed oligopoly market model in which private firms seek to maximize profits and compete with public firms that aim to maximize social welfare. Moreover, Matsumura (1998) considered the objective function of the public firm as a weighted combination of social welfare and profits and showed that partial privatization is optimal for maximizing social welfare. However, although most real-world production activities occur in urban areas and are affected by household activities such as commuting to work, few models explicitly account for the endogenous spatial structure within the monocentric city framework.

Moreover, interregional migration, which represents the movement of labor as an input factor, influences both the urban structure and the production structure. A substantial body of research on interregional migration has accumulated since Lewis (1954) and Harris and Todaro (1970) analyzed the movement of labor between urban and rural regions within a dualistic economy. Although Krugman (1991) and Abdel-Rahman and Fujita (1990), among others, developed models incorporating monopolistic competition, most notably the core–periphery model, they did not consider mixed oligopoly markets. Naito (2012) presented a theoretical model that integrates urban–rural migration with a mixed oligopoly but did not analyze the internal spatial structure of cities themselves, such as the land rent and commuting costs.

Based on the foregoing, in this paper, we construct a general equilibrium model in which the monocentric city framework is incorporated into a mixed duopoly model and analyze how the production efficiency of the public firm affects the urban structure, equilibrium prices, and social welfare. The main results of our analysis are as follows. The impact of a decline in the production efficiency of the public firm on social welfare depends on the trade-off between its direct effect on prices through firm behavior in the mixed oligopoly market and the market size effect arising from the contraction of city size. The results of the comparative statics analysis show that the effect of the production efficiency of the public firm on social welfare is generally ambiguous. Thus, it is necessary to evaluate public firm policy from a general equilibrium perspective that explicitly accounts for the urban structure

The remainder of this paper is organized as follows. Section 2 derives household behavior, the urban structure, and the behavior of public and private firms under the mixed oligopoly as well as the resulting equilibrium. Section 3 presents the results of the comparative statics analysis based on the equilibrium obtained in Section 2 to clarify the effects of the production efficiency of the public firm on social welfare. Finally, Section 4 summarizes the main findings and discusses remaining issues.

2 The model

We consider an open monocentric city model in which the CBD is located at point 0 and the city has a linear spatial structure. Each location is characterized by its distance r from the CBD. The characteristics of all locations are assumed to be identical, except for their distance from the CBD, and one unit of land is supplied at each location. As we consider an open monocentric city model, we assume that the utility enjoyed by households outside the city is given exogenously as $\bar{U}$. All production occurs in the CBD. Households residing in the city are endowed with one unit of labor and commute to the CBD, where they supply labor inelastically to the production sector.

2.1 Households

All households in the city are assumed to have identical preferences and share a common utility function. They derive utility from manufactured goods x and agricultural goods y. We assume that all households share the following quasi-linear utility function:

$$U = ax - \frac{1}{2}x^2 + y \tag{1}$$

where x and y denote the consumption of manufactured and agricultural goods, respectively and a represents reservation demand. Quasi-linear preferences are adopted for several important reasons. First, this approach ensures analytical tractability and allows us to derive closed-form general equilibrium solutions and comparative statics, which are the main contributions of this paper. Second, quasi-linearity eliminates income effects in demand for manufactured goods, thereby isolating the spatial market size mechanism central to our analysis. This simplification enables the transparent decomposition of social welfare into the consumer surplus, producer surplus, land rent, and external utility of households. Finally, the specification is consistent with the urban economics literature and Naito (2012), facilitating a comparison with existing results while extending their scope to incorporate the endogenous urban structure.

Moreover, a household located at distance r from the CBD is assumed to demand one unit of land inelastically. Let $t(> 0)$ denote the per-unit transportation cost. We further assume that the total differential land rent generated within the city is redistributed to the households residing in the city. Under this assumption, the household's budget constraint is given by

$$w + \frac{1}{N}\int_0^{r_f} R(r)dr = px + t + tr + R(r) \tag{2}$$

where w, p, r_f, and $R(r)$ denote the wage rate, price of the manufactured good, urban boundary, and land rent at location r, respectively. By maximizing (1) subject to (2), we obtain the demand function for the manufactured goods of a household located at distance r from the CBD as follows:

$$x = a - p \tag{3}$$

Because the utility function (1) is quasi-linear, there is no income effect. Substituting this result into (2), we can also derive y as follows:

$$y = w + \frac{1}{N}\int_0^{r_f} R(r)dr - R(r) - tr - p(a-p) \tag{4}$$

Substituting (3) and (4) into (1), we obtain the indirect utility function of a household located at distance r from the CBD as follows:

$$V(r) = \frac{1}{2}(a-p)^2 + w + \frac{1}{N}\int_0^{r_f} R(r)dr - R(r) - tr \tag{5}$$

As (3) is linear, the consumer surplus of a household located at r, denoted by $CS(r)$, is given by

$$CS(r) = \frac{1}{2}(a-p)^2. \tag{6}$$

When the number of households in a city is denoted by N, total demand for manufactured goods in the city is given by $Q(p) = N(a-p)$.

2.2 Urban structure

Next, we consider the urban structure. Because we consider a small open city, the equilibrium number of households in the city is determined endogenously, given the exogenously determined level of utility. When exogenous utility is denoted by $\bar{U}$, (5) is equal to $\bar{U}$, that is,

$$\bar{U} = \frac{1}{2}(a-p)^2 + w + \frac{1}{N}\int_0^{r_f} R(r)dr - R(r) - tr \tag{7}$$

Solving (7) for R(r), we obtain the land rent at location r:

$$R(r) = \frac{1}{2}(a-p)^2 + w + \frac{1}{N}\int_0^{r_f} R(s)ds - tr - \bar{U} \tag{8}$$

If we assume that the opportunity cost of land is zero, the land rent at r_f is zero in equilibrium. As all households attain the same utility in equilibrium, equilibrium utility can be written as utility at the urban boundary:

$$0 = \frac{1}{2}(a-p)^2 + w + \frac{1}{N}\int_0^{r_f} R(r)dr - tr_f - \bar{U} \tag{9}$$

By subtracting (8) from (9), the equilibrium land rent function is obtained as

$$R(r) = \begin{cases} t(r_f - r), & r \in [0, r_f], \\ 0, & r > r_f. \end{cases} \tag{10}$$

Let TR denote the total differential land rent generated within the city. Substituting r_f into (Land rent function), we obtain TR as

$$TR \equiv \int_0^{r_f} R(r)\, dr. \tag{11}$$

Because each household is assumed to demand one unit of land inelastically, the urban boundary is given by $r_f = N$. Substituting $r_f = N$ into (10), we obtain the following expression:

$$0 = \frac{1}{2}(a - p)^2 + w - \frac{t}{2}N - \bar{U} \tag{12}$$

Solving (12) for N and using the condition $r_f = N$, we derive the equilibrium number of households and equilibrium urban boundary as follows:

$$N = r_f = \frac{1}{t}\left((a - p)^2 + 2(w - \bar{U})\right) \tag{13}$$

Finally, we conduct comparative statics to examine how t and $\bar{U}$ affect N:

$$\frac{\partial N}{\partial t} < 0, \quad \frac{\partial N}{\partial \bar{U}} < 0$$

2.3 Production

We now turn to the production sector. In this model, both the manufactured goods and the agricultural goods are produced in the CBD and labor is the only input factor. Under perfect competition, it takes one unit of labor to produce one unit of the agricultural good. Moreover, under free labor mobility between the manufactured goods sector and the agricultural sector, the wage rate in the city is normalized to w=1. In the manufactured goods sector, we consider a mixed duopoly market in which a public firm that maximizes social welfare competes with a private firm that maximizes profit. Because these public and private firms are located in the CBD and supply the manufactured good, their outputs are denoted by q_0 and q_1, respectively. Both firms take the urban structure as given and determine their output. As total demand for manufactured goods in the city has already been derived as $Q(p) = N(a - p)$, we obtain the following inverse demand function:

$$p = a - \frac{1}{N}(q_0 + q_1) \tag{14}$$

where q_0 and q_1 represent the output of the public and private firms, respectively. Each firm takes N in (13) as given. Labor is the only input in production. In the manufacturing sector, the cost functions in (15) can be interpreted as arising from production technologies in which the marginal labor requirement for the public firm is θq_0 and that for the private firm is q_1. Next, we specify the cost function of firm i as $C_i(q_i)$:

$$C_i(q_i) = \begin{cases} \frac{\theta}{2} q_i^2 & (i = 0), \\ \frac{1}{2} q_i^2 & (i = 1), \end{cases} \tag{15}$$

where θ is a parameter meaning the production efficiency of the public firm. If $0 < \theta < 1$, the public firm is more efficient than the private firm. By contrast, if $\theta > 1$, the public firm is less efficient than the private firm. When $\theta = 1$, both firms have the same production efficiency. We now define the social welfare function because the public firm chooses its output to maximize social welfare. According to this definition, social welfare is an increasing function of N in the monocentric city model. Therefore, we define social welfare as the sum of the consumer surplus, producer surplus, and total land rent minus $N\bar{U}$. The social welfare function is given by

$$\begin{aligned} W &\equiv CS + PS + \int_0^{r_f} R(r)\, dr - N\bar{U} \\ &= \int_0^Q p(q)\, dq - \sum_{i \in \{0,1\}} C_i(q_i) \; + \int_0^{r_f} R(r)\, dr \; - \; N\bar{U} \\ &= a(q_0 + q_1) - \frac{(q_0 + q_1)^2}{2N} - \frac{\theta}{2} q_0^2 - \frac{1}{2} q_1^2 + \frac{t}{2} N^2 - N\bar{U}. \end{aligned} \tag{16}$$

The profit function of the private firm is given by

$$\pi_1 = \left(a - \frac{1}{N}(q_0 + q_1) \right) q_1 - \frac{1}{2} q_1^2 \tag{17}$$

The public firm maximizes (16) taking the private firm's output q_1 as given, whereas the private firm maximizes (17) taking the public firm's output q_0 as given. [1]

Thus, the first-order conditions for the public and private firms are as follows:

$$\frac{\partial W}{\partial q_0} = a - \frac{q_0 + q_1}{N} - \theta q_0 = 0 \tag{18}$$

$$\frac{\partial \pi_1}{\partial q_1} = a - \frac{q_0 + 2q_1}{N} - q_1 = 0 \tag{19}$$

Solving (18) and (19), we obtain the equilibrium outputs of the public and private firms. Let q_0^* and q_1^* denote the equilibrium outputs of the public and private firms, respectively:

$$q_0^* = \frac{aN(N+1)}{\theta N^2 + (2\theta + 1)N + 1} \tag{20}$$

[1] The agricultural goods market clears automatically under Walras' law, as households allocate their remaining income to the agricultural good, which is produced competitively with one unit of labor per unit of output.

$$q_1^* = \frac{a\theta N^2}{\theta N^2 + (2\theta+1)N + 1} \tag{21}$$

Substituting (20) and (21) into (14), we obtain the equilibrium price of the manufactured goods as follows:

$$p^*(N,\theta) = a\left(1 - \frac{(1+\theta)N + 1}{\theta N^2 + (2\theta+1)N + 1}\right) \tag{22}$$

From (22), it follows that the equilibrium price depends on the demand parameter a, the public firm's production efficiency parameter θ, and the number of households in the city, N. Importantly, whereas a and θ are exogenous parameters, N is endogenously determined through the urban equilibrium. This implies that a change in θ affects the equilibrium price both directly through the firms' strategic behavior in the mixed duopoly and indirectly through its impact on the equilibrium number of households in the city. Accordingly, it is essential to examine both the direct effect of θ and the indirect effect operating through N using comparative statics.

3 Comparative statics

In the preceding section, we described household behavior, the urban structure, and firm behavior and derived the corresponding equilibria. Based on these equilibrium outcomes, in this section, we present the results of comparative statics analysis to examine the effects of the key parameters of the model on social welfare. Although the public and private firms take the number of households in the city as given when determining their outputs in Section 2, the number of households in the city, denoted by N, is endogenously determined in general equilibrium.

3.1 Existence and uniqueness of equilibrium

We first show that the equilibrium number of households in the city is uniquely determined by combining the urban equilibrium condition with the equilibrium outputs in the mixed duopoly market. Substituting (22) into (13), we obtain the following condition that simultaneously satisfies the urban equilibrium and the equilibrium outcomes in the mixed duopoly market:

$$N = \frac{1}{t}\left[a^2\left(\frac{f(N)}{g(N)}\right)^2 + 2(1-\bar{U})\right]. \tag{23}$$

where $f(N)$ and $g(N)$ are defined by $f(N) \equiv (1+\theta)N + 1$ and $g(N) \equiv \theta N^2 + (2\theta+1)N + 1$, respectively. Moreover, we define the following function based on (23):

$$F(N) \equiv tN - \left[a^2\left(\frac{f(N)}{g(N)}\right)^2 + 2(1-\bar{U})\right] \tag{24}$$

Differentiating (24) with respect to N, we obtain the following expression:

$$\frac{dF(N)}{dN} = t - 2a^2\left(\frac{f(N)}{g(N)}\right) \times \frac{d}{dN}\left(\frac{f(N)}{g(N)}\right). \tag{25}$$

After defining $\frac{f(N)}{g(N)}$ as $h(N)$, we investigate the sign of $\frac{d}{dN}\left(\frac{f(N)}{g(N)}\right)$. Differentiating $\frac{f(N)}{g(N)}$ with respect to N,

$$\frac{d}{dN}\left(\frac{f(N)}{g(N)}\right) = -\frac{\theta[(\theta+1)N^2+2N+1]}{[\theta N^2+(2\theta+1)N+1]^2} < 0.$$

Therefore, the function $F(N)$ is strictly increasing in N because the sign of (25) is positive. Next, we confirm the sign of $F(0)$:

$$F(0) = -\left[a^2 + 2(1-\bar{U})\right] < 0.$$

As $F(N) is continuous on [0,\infty)$, satisfies $F(0) < 0$, and $\lim_{N\to\infty} F(N) = +\infty$, the intermediate value theorem implies that there exists a unique $N^* \geq 0$ such that $F(N^*) = 0$. Therefore, we obtain the following proposition.

Proposition 1:
For a, θ, $t > 0$, the equilibrium number of households in the city, N^*, is uniquely determined.

3.2 Impact of the production efficiency of the public firm on city size

Recalling that the urban boundary is given by (1212), city size r_f is also uniquely determined. Thus, $F(N^*) = 0$ is given by

$$tN^* - \left[a^2\left(\frac{f(N^*)}{g(N^*)}\right)^2 + 2(1-\bar{U})\right] = 0 \tag{26}$$

Applying the implicit function theorem to (26) and considering that (25) is positive, we obtain the following expression: [2]

$$\frac{dN^*}{d\theta} = -\frac{\frac{\partial F(N^*)}{\partial\theta}}{\frac{\partial F(N^*)}{\partial N^*}} < 0 \tag{27}$$

From (26) and the fact that the equilibrium urban boundary is given by $r_f = N$, we obtain the following proposition.

[2] $\frac{\partial}{\partial\theta}\left(\frac{(1+\theta)N+1}{\theta N^2+(2\theta+1)N+1}\right) = -\frac{N(N+1)^2}{\left(\theta N^2+(2\theta+1)N+1\right)^2} < 0$. Moreover, the sign of $\frac{\partial F(N^*)}{\partial N^*}$ is positive, as (24) is positive. Consequently, the sign of $\frac{\frac{\partial F(N^*)}{\partial\theta}}{\frac{\partial F(N^*)}{\partial N^*}}$ is negative.

Proposition 2:
An increase in θ reduces the equilibrium number of households in the city and shifts the urban boundary inward.

This proposition implies that a decline in the production efficiency of the public firm moves the urban boundary inward.

3.3 Impact on market prices

Next, we investigate how θ affects the equilibrium price denoted by p^*. As (22) is a function of N and θ, we differentiate (22) with respect to θ as follows:

$$\begin{aligned}
\frac{dp^*}{d\theta} &= \frac{\partial p^*}{d\theta} + \frac{\partial p^*}{dN} \cdot \frac{\partial N^*}{d\theta} \\
&= \frac{a\ N^*(N^*+1)^2}{\left[\theta(N^*)^2 + (2\theta+1)N^* + 1\right]^2} + \frac{a\theta\left[\theta(N^*)^2 + (2\theta+1)N^* + 1\right]}{\left[\theta(N^*)^2 + (2\theta+1)N^* + 1\right]^2} \cdot \frac{dN^*}{d\theta} \\
&\gtreqless 0. \qquad (28)
\end{aligned}$$

The first term on the right-hand side of (28) represents the direct price-increasing effect of the production inefficiency of the public firm, whereas the second term captures the market size effect arising from urban contraction. Although the first term on the right-hand side of (28) is positive, the second term is always negative from (27). Hence, the sign of $\frac{dp^*}{d\theta}$ depends on the relative magnitudes of the positive direct effect of the production inefficiency of the public firm on the equilibrium price and the negative effect operating through the equilibrium number of households.

To clarify the underlying economic mechanism, outputs are strategic substitutes under Cournot competition. Therefore, a contraction in the public firm's output induces an expansion in the private firm's output. However, the increase in the private firm's output is dominated by the reduction in the public firm's output, so that the total market supply decreases and the equilibrium price rises. At the same time, the production inefficiency of the public firm reduces the equilibrium number of households in the city, thereby shrinking the effective market size. This urban contraction lowers total demand and exerts downward pressure on the equilibrium price. Consequently, the equilibrium price reflects the interaction between these two opposing forces: the direct price-increasing effect arising from strategic competition and the indirect price-decreasing effect operating through endogenous changes in the urban structure.

Proposition 3:
An increase in θ has an ambiguous effect on the equilibrium price p^* because the direct price-increasing effect of the production inefficiency of the public firm is offset by the indirect market size effect arising from urban contraction.

3.4 Welfare effects and decomposition

Finally, we examine the impact of the production efficiency of the public firm on social welfare. We derive the equilibrium number of households N^*, urban boundary r_f, equilibrium outputs of the public and private firms (q_0^*,q_1^*), and equilibrium price p^*. We then substitute these into (16) and define social welfare, evaluated at the equilibrium population size, as W^*:

$$W^* = \frac{t}{2}N^2 - N\bar{U} - \frac{a^2}{2}\ \frac{N\left[(N-1)\theta^2(N)^2 + \theta N(N+1) - (N+1)^2\right]}{\left[\theta(N)^2 + (2\theta+1)N + 1\right]^2}. \quad (29)$$

As (29) is a function of θ and N, and N is also a function of θ, the impact of θ on W^* consists of the direct effect of θ and the indirect effect operating through N. Recalling $f(N) \equiv (1+\theta)N + 1$, $g(N) \equiv \theta N^2 + (2\theta+1)N + 1$, the derivative $\frac{dw^*}{d\theta}$, evaluated at N^*, is given by

$$\begin{aligned}
\frac{dW^*}{d\theta}|_{N=N^*} \quad &= \quad -\frac{a^2(N^*)^2(N^*+1)\left[-\theta(N^*)^3 + (\theta+3)(N^*)^2 + 6N^* + 3\right]}{2\left(\theta(N^*)^2 + (2\theta+1)N^* + 1\right)^3} \\
&+ \quad tN^* - \bar{U} - \frac{a^2}{2}\ \frac{\frac{\partial f(N^*)}{\partial N}\ g(N^*) - 2f(N^*)\ \frac{\partial g(N^*)}{\partial N}}{\left[g(N^*)\right]^3} \cdot \frac{\partial N^*}{\partial \theta} \\
&\gtreqless \quad 0. \qquad (30)
\end{aligned}$$

The first term on right-hand side of (30) describes the direct impact of θ on social welfare, whereas the second term on right-hand side of (30) represents the indirect impact of θ through changes in the urban population. Under the direct effect, an increase in θ raises the production cost of the public firm, thereby reducing its profit. This impact means a decline in its production efficiency. Therefore, the sign of the first term on the right-hand side of (30) is generally ambiguous, as the direct impact of a change in θ on social welfare is determined by the relative magnitudes of the reduction in the public firm's profit and consumer surplus and the increase in the private firm's profit. The second term captures the effect of a change in θ on social welfare operating through its impact on the urban population. From (27), it follows that an increase in θ reduces the equilibrium number of households in the city. Although a decline in the number of households in the city reduces the manufactured goods market, it also decreases the total commuting costs to the CBD and the urban cost term $N\bar{U}$. The impact of θ on social welfare is ambiguous, as it depends on the trade-off between the market contraction and the reduction in total commuting costs and urban costs. Therefore, the sign of (30) is determined by the interaction of these effects and is generally ambiguous. Summarizing this discussion, if the reduction in total commuting costs due to the contraction of city size and the associated decrease in external urban costs outweighs the negative market size effect of an increase in θ, social welfare can increase despite the decline in the production efficiency of the public firm.

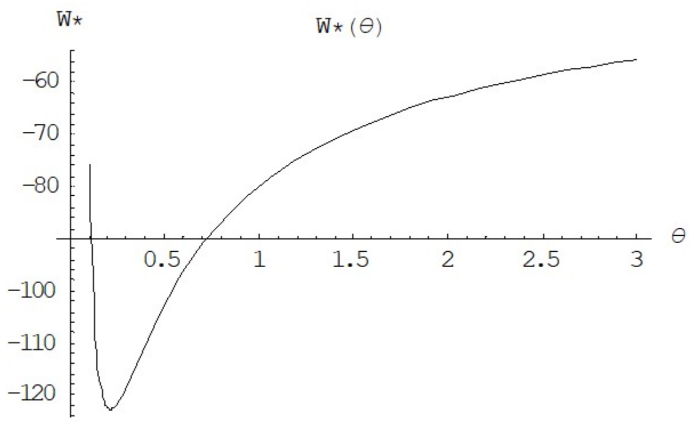

Figure 1: The impact of θ on W^* ($\bar{U} = 2$)

Proposition 4:
An increase in θ, which represents a decline in the production efficiency of the public firm, improves (worsens) social welfare if the reduction in total commuting costs and external urban costs dominates (is dominated by) the negative effect of the contraction in demand.

3.5 Numerical illustration

We present a numerical illustration of the analytical results to clarify the economic mechanisms underlying the price and welfare effects on the production efficiency of the public firm. Equation (30) shows that the effect of θ on social welfare consists of not only a direct effect but also an indirect effect operating through changes in the number of households in the city. Because the number of households in the city is endogenously determined in the small open city framework, one of the most important parameters in this context is exogenously given outside utility $\bar{U}$. We therefore conduct numerical simulations to examine how the impact of θ on social welfare varies with different values of $\bar{U}$. Figures 1–3 illustrate social welfare, denoted by $W^*(\theta)$, for $\bar{U} = 2, 4$, and 6. [3] In all three cases, social welfare initially decreases as θ increases, reflecting the direct market distortion caused by the decline in the production efficiency of the public firm. However, beyond a threshold value of θ, social welfare begins to increase monotonically, indicating that the indirect welfare gains arising from the contraction of the number of households in the city dominate the negative market size effect. The turning point at which the welfare effect switches sign is remarkably stable across different values of $\bar{U}$, whereas overall social welfare

[3] As for other parameters, we fix $a = 15$ and $t = 1$. Moreover, we use $w = 1$ and the range of θ from 0 to 3.

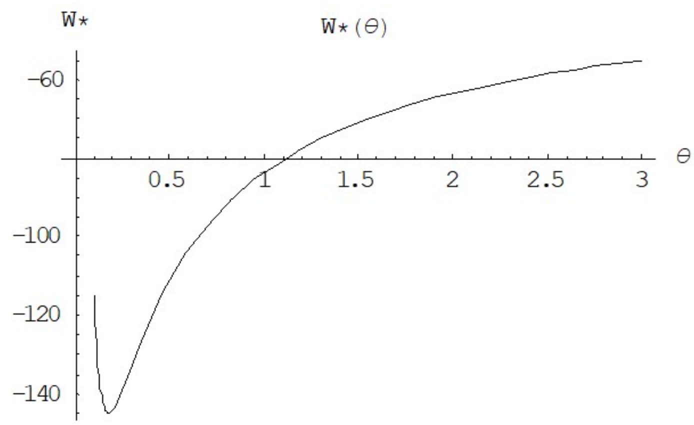

Figure 2: The impact of θ on W^* ($\bar{U} = 4$)

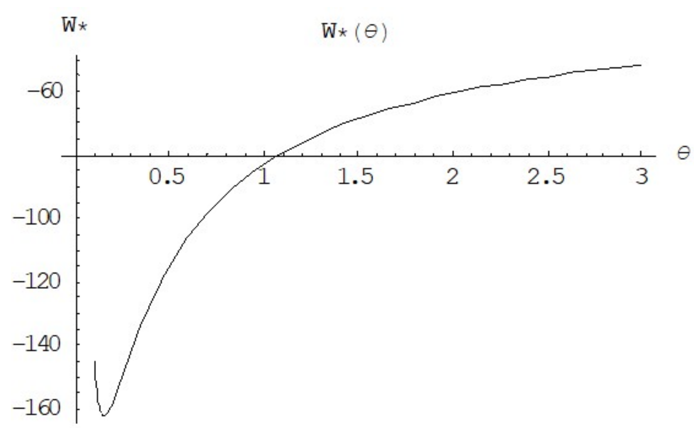

Figure 3: The impact of θ on W^* ($\bar{U} = 6$)

shifts with the outside option. These results confirm the theoretical prediction that the welfare effect of the production inefficiency of the public firm is generally ambiguous and depends on the interaction between the direct and indirect effects.

4 Conclusion

This paper presents a general equilibrium model that integrates a mixed duopoly market into the monocentric city framework to examine how changes in the production efficiency of the public firm affect the urban structure, market outcomes, and social welfare. By explicitly linking firms' strategic behavior with endogenous city size and the urban population, the model provides a unified framework in which industrial organization and urban economics interact. The analysis showed that a decline in the production efficiency of the public firm contracts the equilibrium city size and reduces the urban population. At the same time, the effect of the production inefficiency of the public firm on social welfare is generally ambiguous. Although lower productivity tends to increase production costs and distort market outcomes, it also induces urban contraction, which reduces total commuting and external urban costs. Therefore, social welfare may increase when spatial cost reductions dominate the negative market size effect. This result highlights the importance of evaluating public firm policies within a general equilibrium framework that explicitly considers the urban structure. These findings suggest that conventional evaluations of the production efficiency of the public firm, which focus solely on production-side distortions, may be misleading when considering the spatial structure and household behavior. Policies affecting the production efficiency of the public firm can have significant indirect effects through the urban form and these spatial feedback mechanisms are crucial for assessing social welfare. Several extensions remain for future research. First, the model abstracts from partial privatization, even though many public firms operate under mixed ownership structures. Incorporating the degree of privatization into the analysis would allow for a richer evaluation of public firm reform. Second, the present model assumes that households consume land in fixed quantities. Extending the framework to allow for endogenous land consumption, as in the more general formulations of Alonso (1964) and subsequent studies, would provide further insights into the interaction between the market structure and urban form.

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